Exam Waves and Optics – 30 January 2014

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This exam contains 6 questions on 4 pages.

A few **preliminary remarks**:

- Answers may be given in Dutch.
- Use a new sheet of paper for each question.
- Put your name and student number at the top of all sheets.
- Put your student card at the edge of the desk for checking by the assistants and show it when handing in your exam.

Question 1 (10 points). Energy transport by an electromagnetic wave

The energy density (unit: J/m^3) of an electric field of magnitude E, in vacuum, is given by:

$$u_E = \frac{\varepsilon_0}{2} E^2$$

The energy density (unit: J/m^3) of a magnetic field of magnitude B, in vacuum, is given by:

$$u_B = \frac{1}{2\mu_0} B^2$$

(ε_0: electric permittivity of the vacuum; μ_0: magnetic permeability of the vacuum)

Questions:

a. Show, based on the properties of an electromagnetic wave, that the energy densities of the electric and magnetic field associated with an electromagnetic wave are equal: $u_E = u_B$.

b. Derive an expression for the so-called Poynting-vector which describes the direction and magnitude of the (instantaneous) transport of energy per unit time and unit surface area, expressed in J/(s m^2) or W/m^2, of an electromagnetic wave.
Hint: Consider the energy flowing during a time interval Δt through a surface area A.

(Question 2 on the next page)

Question 2 (6 points): Wave functions

Which of the following expressions represents a wave function ? Explain how you reach your conclusion.

a) $\psi(x,t) = (2x - 3t)^2$

b) $\psi(x,t) = (6x + 2t + 5)^2$

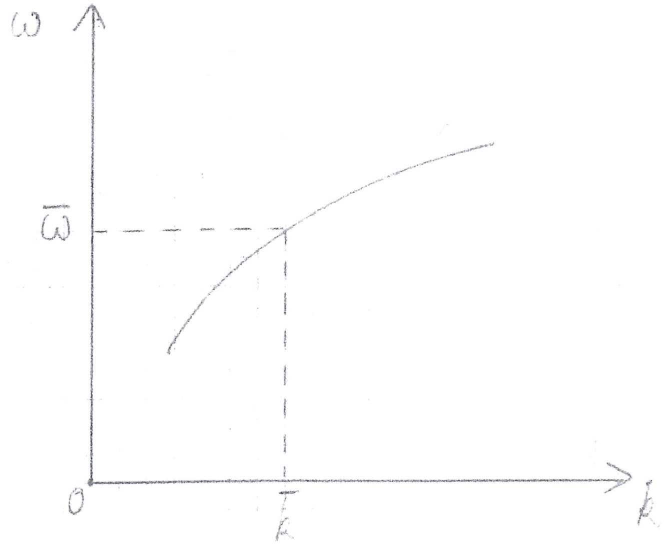
c) $\psi(x,t) = 1/(x^2 + 3t)$

x represents the spatial coordinate (unit: m)
t represents time (unit: s)

For those representing a wave function, what is the size and direction of the velocity of the wave ?

Question 3 (4 points): The dispersion relation

In the description of the superposition of harmonic waves with different frequencies, the concepts of phase velocity and group velocity are used. The dispersion relation relates the angular frequency ω of a wave to its propagation number k: $\omega(k)$. Making use of the dispersion relation, a graphical representation of the phase and group velocities was discussed during the lectures. Below an example of a dispersion relation.

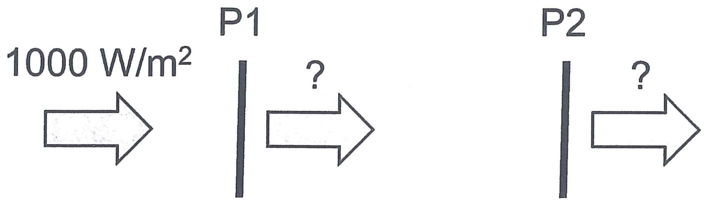


Question:
Indicate in this figure (make a copy of it in your answer) the graphical representation of the phase velocity and the group velocity of a superposition of harmonic waves with average angular frequency $\overline{\omega}$ and average propagation number $\overline{k}$.

(Question 4 on the next page)

Question 4 (6 points): Linear polarizers

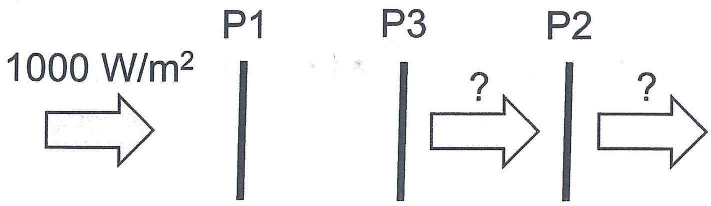
Natural light with an irradiance of 1000 W/m^2 impinges on two consecutive linear polarizers P1 and P2 (see figure below).



Polarizer P1 has a direction of polarization of +20 degrees relative to the vertical, polarizer P2 has a direction of polarization of +50 degrees relative to the vertical.

Question a:
What is the irradiance in W/m^2 of the light after P1 and after P2 ?

Now consider that a third linear polarizer P3, with a direction of polarization of –25 degrees relative to the vertical, is placed between P1 and P2 (see figure below).



Question b:
What is the irradiance in W/m^2 of the light after P3 and after P2 ?

Note: The polarization direction relative to the vertical is expressed by a positive or negative number. The '+' and '–' sign indicate opposite directions relative to the vertical: clockwise and counterclockwise.

Question 5 (8 points): Reflection from a thin film

Consider a layer of water (index of refraction 1.33) with a thickness of 600 nm deposited on a glass plate (index of refraction 1.50). Above the water is air. The film is illuminated from above with sunlight.

Question:
Which vacuum wavelengths within the visible spectrum (300 to 800 nm) are suppressed due to interference in the reflected light ?

(Question 6 on the next page)

Question 6 (6 points): The Huygens-Fresnel principle

In the lectures, the Huygens-Fresnel principle was used to deduce/study the propagation of waves and their diffraction at apertures. The Huygens-Fresnel principle states:

> Every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency as the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets.

The Huygens-Fresnel principle as stated above has a shortcoming that is solved by the introduction of the inclination (or obliquity) factor.

Questions:

a) Which problem of the Huygens-Fresnel principle does the inclination factor solve ?

b) What is the inclination factor (explain in words and illustrate with a drawing) ?

Even taking the inclination factor into account, a remaining shortcoming of the Huygens-Fresnel principle was pointed out during the lectures.

Question:

c) Which shortcoming of the Huygens-Fresnel principle remains after taking the inclination factor into account ?